Flow Around a Parabola

Consider a parabola whose apex is the origin of coordinates O, and and whose axis of symmetry is the x axis. The equation for such a parabola is given by

$$y^2 - 2Rx = 0, \tag{1}$$

where R is the radius of curvature at the apex. The plane of the parabola will be referred to as the z-plane, where $z = x + iy$. We transform the parabola into the ξ axis of the ζ-plane, where $\zeta = \xi + i\eta$, using the conformal mapping

$$z = \frac{\zeta^2}{2R} + i\zeta. \tag{2}$$

Note that

$$\zeta = -iR + \sqrt{2Rz - R^2}. \tag{3}$$

The branch cut in (3) is defined along the parabola. From (2) we have

$$\frac{dz}{d\zeta} = \frac{\zeta + iR}{R} = \sqrt{\frac{2z}{R} - 1}. \tag{4}$$

We now consider the following two cases:

1. **Uniform Flow U_∞ at $x \to -\infty$**

   In the ζ-plane, we take the stagnation flow

   $$(u - iv)_\zeta = \frac{V\zeta}{R}, \tag{5}$$

   where V is a constant to be determined. The velocity in the physical plane is

   $$(u - iv)_z = \frac{V\zeta}{\zeta + iR} = V(1 - \frac{i}{\sqrt{\frac{2z}{R} - 1}}). \tag{6}$$

   As $x \to -\infty$, $u \to V = U_\infty$. Along the negative x axis,

   $$u = U_\infty(1 - \frac{1}{\sqrt{1 - \frac{2x}{R}}}), \tag{7}$$

   and

   $$\frac{du}{dx} = -\frac{U_\infty}{R}\frac{1}{(1 - \frac{2x}{R})^{\frac{3}{2}}}. \tag{8}$$

2. **Flow Turning Around a Parabola**

In the ζ-plane, we take

$$(u - iv)_\zeta = V, \tag{9}$$

where V is a constant to be determined. The velocity in the physical plane is

$$(u - iv)_z = \frac{VR}{\zeta + iR} = \frac{V}{\sqrt{\frac{2z}{R} - 1}}. \tag{10}$$

At the apex, the stagnation point, $u_0 = 0$, and $V = v_0$. Therefore,

$$(u - iv)_z = \frac{v_0}{\sqrt{\frac{2z}{R} - 1}}. \tag{11}$$